A – 1- كتابة العدد $(\sqrt{3}-i)^2$ على الشكل الجبري

$$(\sqrt{3}-i)^2=\left(\sqrt{3}\right)^2-2\sqrt{3}i+i^2$$
$$=3-2\sqrt{3}i-1$$

وبالتالي فإن : $\left(\sqrt{3}-i\right)^2=2-2\sqrt{3}i$

2- حل المعادلة $z^2-\left(\sqrt{3}+3i\right)z+2\sqrt{3}i-2=0$

المميز المختصر لهذه المعادلة هو : $\Delta=\left(\sqrt{3}+3i\right)^2-4(2\sqrt{3}i-2)$

$$=3+6\sqrt{3}i-9-8\sqrt{3}i+8=2-2i\sqrt{3}$$
$$=(\sqrt{3}-i)^2$$

إذن حلاها هما :

$$z_1=\frac{\sqrt{3}+3i-\sqrt{3}+i}{2}=2i$$

و $$z_2=\frac{\sqrt{3}+3i+\sqrt{3}-i}{2}=\sqrt{3}+i$$

إذن مجموعة حلول المعادلة هي : $\left\{2i,\sqrt{3}+i\right\}$

B – 1- أ- كتابة z_1 و z_2 على الشكل المثلثي

بما أن : $z_1=2i$

$$=2(0+1i)=2\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$$

فإن : $z_1=\left[2,\frac{\pi}{2}\right]$

وبما أن : $z_2=\sqrt{3}+i$

$$=2\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)=2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$$

فإن : $z_2=\left[2,\frac{\pi}{6}\right]$

ب- التحقق من أن : $z_1^{12}=z_2^{12}$

$$z_1^{12}=\left[2,\frac{\pi}{2}\right]^{12}$$
$$=\left[2^{12},12.\frac{\pi}{2}\right]=\left[2^{12},6\pi\right]$$
$$=\left[2^{12},0\right]=2^{12}(\cos 0+i\sin 0)=2^{12}$$

وبما أن : $z_2^{12}=\left[2,\frac{\pi}{6}\right]^{12}$

$$=\left[2^{12},12,\frac{\pi}{6}\right]=\left[2^{12},2\pi\right]=\left[2^{12},0\right]$$
$$=2^{12}$$

فإن : $z_1^{12} = z_2^{12}$

2-أ- كتابة $\frac{z_3}{z_2}$ على الشكل الجبري

بما أن :
$$\frac{z_3}{z_2} = \frac{\sqrt{2}(1+i)}{\sqrt{3}+i}$$
$$= \frac{(\sqrt{2}+i\sqrt{2})(\sqrt{3}-i)}{(\sqrt{3}+i)(\sqrt{3}-i)} = \frac{\sqrt{6}-i\sqrt{2}+i\sqrt{6}+\sqrt{2}}{3+1}$$

فإن :
$$\frac{z_3}{z_2} = \frac{\sqrt{6}+\sqrt{2}}{4} + i\frac{\sqrt{6}-\sqrt{2}}{4}$$

*** كتابة $\frac{z_3}{z_2}$ على الشكل المثلثي**

لدينا :
$$\frac{z_3}{z_2} = \frac{\sqrt{2}+\sqrt{2}i}{\sqrt{3}+i}$$
$$= \frac{2\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)}{\sqrt{3}+i} = \frac{\left[2,\frac{\pi}{4}\right]}{\left[2,\frac{\pi}{6}\right]} = \left[\frac{2}{2},\frac{\pi}{4}-\frac{\pi}{6}\right]$$

إذن :
$$\frac{z_3}{z_2} = \left[1,\frac{\pi}{12}\right]$$

ب- استنتاج $\cos\frac{\pi}{12}$ و $\sin\frac{\pi}{12}$

لدينا :
$$\frac{z_3}{z_2} = 1\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right) = \frac{\sqrt{6}+\sqrt{2}}{4} + i\frac{\sqrt{6}-\sqrt{2}}{4}$$

إذن :
$$\cos\frac{\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

و
$$\sin\frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

3-أ- لنبين أن O هي مركز الدائرة المحيطة بالمثلث ABC

سنبين أن OA = OB = OC

لدينا :
$$OA = |z_1 - 0|$$
$$= |z_1| = 2$$

و
$$OB = |z_2 - 0|$$
$$= |z_2| = 2$$

و
$$OC = |z_3 - 0|$$
$$= |z_3| = |\sqrt{2}||1+i| = \sqrt{2}.\sqrt{2} = 2$$

إذن : OA = OB = OC

ب- تحديد $\left(\overrightarrow{OB},\overrightarrow{OC}\right)$

$$\left(\overrightarrow{OB},\overrightarrow{OC}\right) \equiv \arg\frac{z_3-0}{z_2-0}[2\pi]$$

$$\equiv \arg\frac{z_3}{z_2}[2\pi]$$

إذن : $\left(\overrightarrow{OB},\overrightarrow{OC}\right) \equiv \frac{\pi}{12}[2\pi]$